

Calculus Problem Solving: A Review of Krulick and Rudnick's Theory for First Year Engineering Students.

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ABSTRACT

The purpose of this study is to determine the initial ability of Engineering students in working on calculus problems seen from the perspective of Krulick And Rudnick's theory. The research method used is descriptive qualitative involving 151 mechanical engineering students at one of the universities in the city of Semarang. Based on these 151 students, they are classified into several sections focused on the aspects of Read And Think; Explore And Plan; Select A Strategy; Find And Answer, Reflect And Extend. Based on these aspects, there are fundamental weaknesses in the aspects of Explore And Plan; Select A Strategy; Find And Answer, Reflect And Extend. For teachers of calculus material, it is necessary to pay attention to the theoretical aspects of Krulick And Rudnick starting from the explore and plan stage to the reflect and extend aspect.

Keywords: problem solving, krulick rudnick theory, calculus.

INTRODUCTION

Basic calculus is a fundamental subject in engineering education, serving as a core foundation for understanding advanced topics such as differential equations, control systems, optimization, and engineering modeling(Noto et al., 2019). Mastery of basic calculus concepts—such as limits, derivatives, and integrals—is essential for engineering students to develop analytical thinking and problem-solving skills required in their respective disciplines(Priss, 2020). However, numerous studies and classroom observations indicate that many engineering students demonstrate weak performance in solving basic calculus problems.

This weakness is reflected in students' difficulties in understanding abstract mathematical concepts, translating real-world problems into mathematical models, and applying appropriate calculus procedures systematically(Siswono et al., 2020). Common issues include misconceptions about fundamental principles, limited procedural fluency, and an overreliance on memorization rather than conceptual

understanding. As a result, students often struggle when faced with non-routine problems that require higher-order thinking and conceptual reasoning(Billion, 2023).

Several factors contribute to this problem, including students' diverse educational backgrounds, negative perceptions of mathematics, traditional lecture-centered teaching methods, and a lack of contextual or application-based learning relevant to engineering fields(Köğçe et al., 2010). Moreover, the transition from secondary education to higher education often exposes gaps in students' prior mathematical knowledge, which are further exacerbated by the fast-paced nature of calculus instruction in engineering programs(Tarasenkova, N., Kovalenko, 2015).

Although previous studies have investigated students' difficulties in learning calculus and have proposed various instructional strategies, most of these studies focus on general mathematics education or descriptive analyses of learning obstacles(Dahiana et al., 2023). There is still limited empirical research that specifically examines engineering students' weaknesses in basic calculus from a problem-solving perspective that integrates conceptual understanding, procedural fluency, and application in authentic engineering contexts(Erbilgin & Gningue, 2023). Furthermore, few studies have systematically explored instructional interventions or learning models that are explicitly designed to address these weaknesses using technology-enhanced, student-centered, or context-based approaches tailored to engineering education.

Therefore, a clear research gap exists in the need for a comprehensive and empirically validated approach that not only identifies the nature of engineering students' difficulties in basic calculus but also proposes and evaluates effective learning strategies to improve their calculus problem-solving abilities. Addressing this gap is essential to strengthening the quality of engineering education and ensuring that students acquire the mathematical competencies required for success in their academic and professional careers.

METHOD

This study adopted a qualitative descriptive research design to examine engineering students' problem-solving processes in calculus. The qualitative approach was employed to capture students' cognitive strategies, reasoning patterns, and conceptual difficulties beyond mere solution accuracy, thereby enabling an in-depth analysis of how calculus problems are interpreted, planned, and solved.

The participants comprised 151 undergraduate engineering students who had completed a basic calculus course at a higher education institution. A total sampling technique was applied, involving all students enrolled in the selected classes, to ensure comprehensive representation of problem-solving abilities. The relatively homogeneous academic

background of the participants supported meaningful qualitative comparisons across problem-solving levels.

Data were collected using an open-ended calculus problem-solving test specifically designed to elicit students' reasoning in accordance with the Krulick and Rudnick problem-solving framework, which classifies problem-solving performance into four levels: *recall*, *basic*, *critical*, and *creative*. Students' written responses constituted the primary data source for qualitative analysis.

Students' solutions were systematically examined and categorized based on the indicators of the Krulick and Rudnick framework. To enhance analytical depth and credibility, semi-structured interviews were conducted with purposively selected students representing each problem-solving level. These interviews provided additional insight into students' strategic choices, misconceptions, and difficulties encountered during calculus problem solving.

Data analysis followed an interactive qualitative procedure, involving data reduction, data display, and conclusion drawing. During data reduction, students' responses were coded and classified according to problem-solving levels and recurring cognitive patterns. The data were subsequently organized into descriptive narratives and categorical summaries to elucidate distinguishing characteristics across levels. Conclusions were drawn by identifying dominant reasoning patterns and persistent conceptual obstacles in engineering students' calculus problem solving.

The trustworthiness of the findings was ensured through methodological triangulation between written responses and interview data, source triangulation across problem-solving categories, and peer debriefing with experts in mathematics education. Ethical principles were upheld by securing voluntary participation, preserving participant anonymity, and restricting data use to academic purposes.

FINDINGS AND DISCUSSION

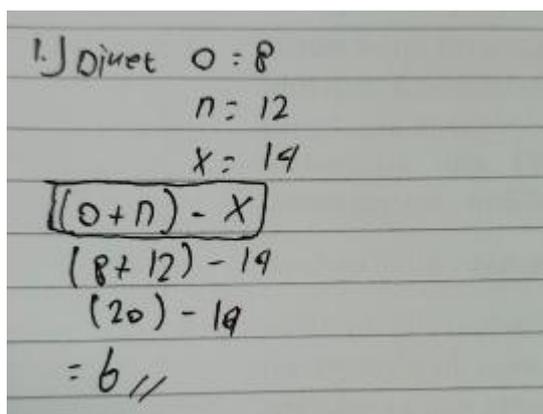
The analysis of written responses from 151 engineering students revealed substantial variation in calculus problem-solving performance across the four levels proposed by Krulick and Rudnick: *recall*, *basic*, *critical*, and *creative*. The majority of students demonstrated problem-solving behaviors concentrated at the recall and basic levels, while considerably fewer students reached the critical and creative levels. This distribution indicates that most students relied heavily on memorization of formulas and routine procedures rather than engaging in higher-order reasoning, strategic planning, and conceptual integration.

Across all levels, recurring difficulties were identified in understanding problem statements, selecting appropriate solution strategies, and justifying mathematical reasoning. These difficulties suggest that students' challenges in

calculus extend beyond computational skills and are closely linked to deficiencies in problem interpretation and conceptual reasoning.

At the recall level, students primarily exhibited difficulties related to conceptual memory and symbol recognition. Many students were able to recall isolated formulas—such as derivative rules or basic integral forms—but failed to recognize when and how these formulas should be applied. Errors commonly observed included incorrect substitution of variables, misinterpretation of mathematical symbols, and confusion between related concepts (e.g., derivatives versus differentials).

Students at this level tended to approach calculus problems as tasks requiring direct retrieval of memorized information rather than as problems requiring interpretation. Interview data confirmed that these students often searched for “the right formula” immediately after reading the question, without first analyzing the underlying mathematical structure of the problem. This behavior indicates a fragmented understanding of calculus concepts, where knowledge is stored as disconnected facts rather than as an integrated conceptual framework. The difficulties on **Figure 1**



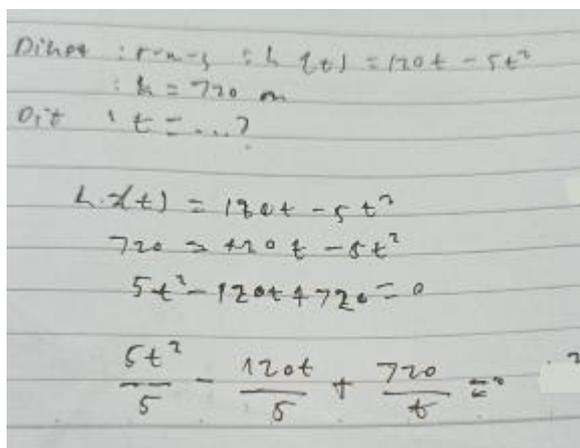
$$\begin{aligned}
 &1.) \text{ Ditet } 0 = 8 \\
 &\quad n = 12 \\
 &\quad x = 14 \\
 &\boxed{(0+n) - x} \\
 &(8+12) - 14 \\
 &(20) - 14 \\
 &= 6 //
 \end{aligned}$$

Figure 1. Difficulties related to conceptual memory and symbol recognition

Students categorized at the basic level demonstrated the ability to follow standard procedures when problems closely resembled examples encountered during instruction. However, significant difficulties emerged when tasks required procedural adaptation or multi-step reasoning. Common errors included incorrect sequencing of solution steps, partial completion of procedures, and failure to connect intermediate results to the overall problem goal.

A notable difficulty at this level was students’ limited ability to translate verbal or contextual problems into mathematical representations. Many students struggled to construct appropriate mathematical models from word problems, resulting in correct procedural execution applied to incorrect or incomplete representations. This indicates that while

procedural knowledge was present, it was not supported by strong representational competence. The representation can see on **Figure 2**



Diker : $r = a - g$; $h(t) = 120t - 5t^2$
 ; $h = 720$ m
 Ditanya : $t = \dots ?$

 $h(t) = 120t - 5t^2$
 $720 = 120t - 5t^2$
 $5t^2 - 120t + 720 = 0$

 $\frac{5t^2}{5} - \frac{120t}{5} + \frac{720}{5} = 0$

Figure 2. Contextual problems into mathematical representations

From a cognitive perspective, these findings suggest that students at the basic level operate within a narrow procedural comfort zone and experience difficulty when problems deviate from familiar formats.

At the critical level, students exhibited more advanced reasoning skills, including the ability to analyze problems, select relevant strategies, and monitor their solution processes. Nevertheless, difficulties were still evident in evaluating solution efficiency and verifying results. Some students arrived at correct solutions but were unable to provide logical justifications or explain why their methods were valid.

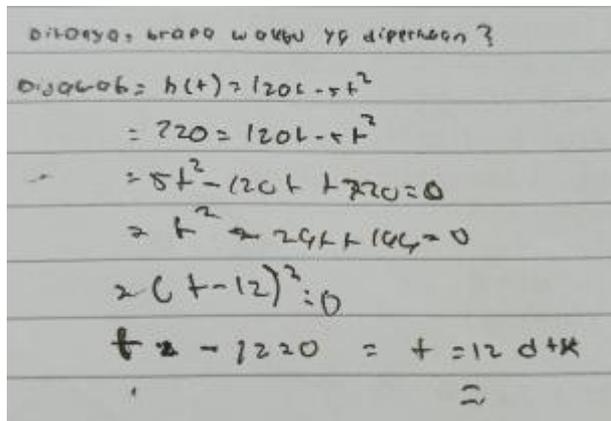
Interview data revealed that students at this level often lacked confidence in their reasoning, leading them to second-guess correct solutions or abandon effective strategies prematurely. This suggests that metacognitive regulation—particularly self-evaluation and reflection—remains underdeveloped, even among students demonstrating relatively high problem-solving competence.

These findings indicate that progression to the critical level does not automatically guarantee mastery of reflective and evaluative skills, which are essential components of expert problem solving in calculus.

Only a small proportion of students reached the creative level, characterized by flexible thinking, strategy generation, and the ability to approach problems from multiple perspectives. Even within this group, difficulties emerged in generalizing solutions and transferring strategies to novel contexts. Some students were able to produce innovative solutions for specific problems but struggled to articulate broader principles or apply similar strategies to different problem types.

This limitation suggests that creative problem solving in calculus requires not only originality but also deep conceptual understanding and

abstraction skills. The scarcity of students at this level highlights a systemic issue in calculus instruction, where opportunities to engage in open-ended, exploratory problem solving are limited. The condition can be seen on **Figure 3**



$$0.0000 = 6000 \text{ w } 0.06 \text{ y } \text{ dipertahan?}$$

$$0.0000 = h(t) = 120t - t^2$$

$$= 720 = 120t - t^2$$

$$\rightarrow 5t^2 - 120t + 720 = 0$$

$$= t^2 - 24t + 144 = 0$$

$$2(t-12)^2 = 0$$

$$t = 12 \text{ dan } t = 12$$

Figure 3. Difficulties emerged in generalizing solutions

The dominance of students operating at the recall and basic levels of the Krulick and Rudnick framework indicates that calculus instruction in engineering programs remains largely procedure-oriented (Dahiana et al., 2023). While procedural fluency is necessary, the findings demonstrate that it is insufficient for developing the problem-solving competencies required in engineering practice. (Erbilgin & Gningue, 2023) Engineering problems are inherently complex, ill-structured, and contextualized; therefore, calculus instruction must move beyond repetitive algorithmic exercises toward conceptually driven and strategy-oriented learning experiences (Radford et al., 2019) (Demir et al., 2018).

Instructionally, this implies a need to redesign calculus courses so that formulas and procedures are introduced as tools derived from concepts, rather than as isolated rules to be memorized. Conceptual explanations, graphical interpretations, and physical or engineering-based analogies should be systematically embedded to strengthen students' recall-level understanding and prevent fragmented knowledge structures (Rocha, 2019) (Rohaeti et al., 2019).

The persistent difficulties observed at the basic level, particularly in translating word problems into mathematical representations, highlight a critical gap between calculus instruction and engineering application (Suryaningrum et al., 2020). Many students were able to execute procedures correctly but failed to construct appropriate models, suggesting that modeling competence is underdeveloped (Arzarello & Sabena, 2011) (Weber, 2001).

To address this issue, calculus instruction in engineering education should explicitly integrate mathematical modeling tasks, where students are required to identify variables, formulate assumptions, and represent

real-world engineering situations mathematically. Instructional strategies such as problem-based learning (PBL) and context-rich problems can help students bridge the gap between abstract calculus concepts and engineering contexts. This approach supports the transition from basic to critical problem-solving levels by emphasizing sense-making rather than routine execution.

CONCLUSION

Overall, the findings indicate that engineering students' difficulties in calculus are hierarchical and cumulative. Weaknesses at lower levels—such as poor conceptual understanding and limited representational skills—constrain students' ability to progress toward higher-order problem-solving levels. The dominance of recall- and basic-level behaviors reflects an instructional emphasis on procedural fluency rather than on conceptual reasoning and strategic thinking.

From the perspective of the Krulick and Rudnick framework, these results underscore the importance of instructional designs that explicitly foster progression across problem-solving levels. Enhancing students' calculus performance requires pedagogical interventions that promote conceptual understanding, metacognitive awareness, and creative reasoning, rather than sole reliance on procedural practice.

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