Students' Responses to the Implementation of a Mathematical Problem-Solving Model Based on Mathematization and Mathematical Modeling in Integral Calculus

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ABSTRACT

Mathematical problem-solving is an essential competency in mathematics education, particularly in Integral Calculus, which requires students to understand problems, construct mathematical representations, develop mathematical models, interpret results, and reflect on solution processes. Although mathematical problem-solving models have been widely implemented, studies investigating students' responses to each stage of the model remain limited. This study aimed to describe students' responses to the implementation of a mathematization- and mathematical modeling-based problem-solving model in Integral Calculus. A quantitative descriptive design was employed involving 37 undergraduate students in the Mathematics Education Study Program at Universitas Muhammadiyah Semarang, Indonesia. Data were collected using a questionnaire based on the model's five stages and analyzed using descriptive statistics. The findings showed that the model received a very positive response, with an overall mean score of 4.42. The highest response was observed in the Understanding the Problem and Identifying Information stage (M = 4.58), whereas Constructing Initial Representations received the lowest mean score (M = 4.14). Students' responses improved in the subsequent stages, indicating that mathematization remained the most challenging phase, while reflection was perceived very positively. These findings provide empirical evidence that analyzing students' responses across each stage offers valuable insights for evaluating and improving the implementation of mathematical problem-solving models in Integral Calculus.

Keywords: Integral Calculus; Mathematical Modeling; Mathematical Problem-Solving; Mathematization; Student Responses

INTRODUCTION

Mathematical problem-solving is one of the core competencies in mathematics education, playing a fundamental role in fostering mathematical reasoning, communication, representation, connections, and decision-making. This competency extends beyond obtaining correct answers; it encompasses understanding problems, constructing mathematical representations, selecting appropriate solution strategies, interpreting results, and reflecting on the solutions obtained (Polya, 1973; Purnomo et al., 2022, 2024; Schoenfeld, 1987, 1992). Consistent with the demands of twenty-first-century education, recent research has increasingly conceptualized problem-solving not merely as a learning outcome but as a complex cognitive process involving mathematical thinking, self-regulation, and reflective reasoning throughout the problem-solving process. Consequently, mathematical problem-solving has become one of the primary indicators for evaluating the quality of mathematics instruction across educational levels.

Within higher education, mathematical problem-solving is particularly essential in Integral Calculus, a course that requires students not only to master integral concepts and computational procedures but also to understand problem situations, construct mathematical representations, formulate mathematical models, select appropriate solution strategies, interpret results, and evaluate the validity of solutions within the given context. These demands make Integral Calculus one of the mathematics courses that requires advanced mathematical thinking. Previous studies have reported that undergraduate students continue to experience difficulties in transforming contextual problems into mathematical models, employing multiple representations appropriately, and connecting mathematical concepts with real-world situations (Purnomo et al., 2014; Purnomo & Mawarsari, 2014; Sulistyaningsih et al., 2021). These findings suggest that students' difficulties are rooted not only in procedural knowledge but also in the underlying cognitive processes involved in mathematical problem-solving.

Various mathematical problem-solving models have been developed to support students in approaching problems systematically. In general, these models emphasize understanding the problem, planning solution strategies, implementing the solution, and evaluating the results obtained (Polya, 1973; Purnomo et al., 2022, 2024; Schoenfeld, 1987, 1992). As mathematics education research has evolved, greater attention has been devoted not only to procedural stages but also to mathematical representation, mathematization, interpretation, and reflection as integral components of problem-solving activities. Based on these theoretical foundations, the present study implemented a mathematical problem-solving model consisting of five stages: (1) Understanding the Problem and Identifying Information, (2) Constructing Initial Representations, (3) Developing Mathematical Models, (4) Interpreting and Validating Results, and (5) Reflection and Revision. This model was designed to guide students through a systematic thinking process, beginning with understanding the problem context and culminating in evaluating the appropriateness of the proposed solution.

Although research on the implementation of mathematical problem-solving models has grown considerably, most previous studies have focused primarily on their effects on problem-solving ability, learning achievement, higher-order thinking skills, or students' academic performance (Purnomo, 2019; Purnomo & Rohman, 2015; Purnomo & Faturohman, 2014). The effectiveness of these models has generally been evaluated using achievement tests or measures of problem-solving performance administered after instruction. By contrast, studies investigating how students respond to each stage of a mathematical problem-solving model remain limited. Yet students' responses constitute an important indicator of implementation quality because they reflect learning experiences, model acceptability, usability, and students' perceptions of each stage throughout the problem-solving process. Such information is essential for identifying which stages have been implemented effectively and which require further refinement.

Based on the foregoing discussion, a significant research gap remains. Existing studies have predominantly evaluated the success of mathematical problem-solving models in terms of improvements in problem-solving performance or learning outcomes, whereas research examining students' responses to each stage of the implementation process is still scarce, particularly within Integral Calculus courses in higher education. Consequently, limited empirical evidence is available regarding how students experience each stage of the problem-solving process and which stages represent the most critical challenges during implementation. Such evidence is essential for evaluating the quality of model implementation and informing subsequent refinements of the model.

Unlike previous studies, the present research evaluates the implementation of a mathematical problem-solving model by examining students' responses to each of its five stages rather than relying solely on learning outcomes or problem-solving performance. This stage-by-stage analysis enables the identification of phases that receive the most positive responses as well as those that continue to present challenges for students throughout the problem-solving process. Accordingly, this study is expected to contribute theoretically by providing empirical evidence regarding students' responses across different stages of mathematical problem-solving and to contribute practically by informing the refinement of problem-solving model implementation in Integral Calculus instruction (Özgeldi & Aydın, 2021). Therefore, this study aims to describe students' responses to the implementation of a mathematical problem-solving model in Integral Calculus based on its five implementation stages. The findings are expected to provide empirical evidence regarding the quality of implementation at each stage, identify stages requiring further improvement, and serve as a basis for enhancing the effectiveness of mathematical problem-solving models in Integral Calculus instruction.

METHOD

This study employed a quantitative descriptive research design to examine students' responses to the implementation of a mathematical problem-solving model in Integral Calculus. A descriptive design was adopted because the study aimed to describe students' response patterns across each stage of the proposed problem-solving model rather than examine relationships or causal effects among variables. The study was conducted with undergraduate students enrolled in the Mathematics Education Study Program at Universitas Muhammadiyah Semarang during the second semester of the 2025/2026 academic year. A total of 37 students participated in the study, and a total sampling technique was employed, whereby all students enrolled in the course were included as research participants.

The mathematical problem-solving model was implemented through five sequential stages: (1) Understanding the Problem and Identifying Information, (2) Constructing Initial Representations, (3) Developing Mathematical Models, (4) Interpreting and Validating Results, and (5) Reflection and Revision. These stages were integrated throughout the Integral Calculus course. Following the implementation, students completed a response questionnaire specifically developed based on the five stages of the proposed model. The questionnaire was designed to ensure *construct alignment* between the theoretical framework of the model and the constructs measured.

The research instrument consisted of a student response questionnaire comprising 42 items representing 21 indicators, including 27 positively worded statements and 15 negatively worded statements. Responses were measured using a five-point Likert scale ranging from 1 (Strongly Disagree) to 5 (Strongly Agree). Scores for negatively worded items were reverse-coded before analysis. Prior to data collection, the instrument underwent content validation by experts in mathematics education and educational assessment. Construct validity and reliability analyses were subsequently conducted to ensure that the instrument consistently measured the intended constructs and was suitable for research purposes.

The data were analyzed using descriptive statistics to describe the observed response patterns (Creswell, 2014; Sugiyono, 2018; Sukestiyarno, 2020). Mean scores were interpreted using five response categories: very low (1.00–1.80), low (1.81–2.60), moderate (2.61–3.40), high (3.41–4.20), and very high (4.21–5.00). The findings are presented through tables and narrative descriptions to identify the stages that received the most positive responses as well as those requiring further improvement, thereby providing a comprehensive evaluation of the implementation of the mathematical problem-solving model in Integral Calculus. Participation was voluntary, and all participant data were treated confidentially and used solely for research purposes.

FINDINGS AND DISCUSSION
FINDINGS

Students' responses to the implementation of the mathematical problem-solving model were analyzed to examine the level of acceptance of each stage of the model implemented in the Integral Calculus course. The analysis was based on questionnaire responses collected from 37 undergraduate students after the completion of the instructional intervention. The questionnaire was developed according to the five stages of the mathematical problem-solving model: (1) Understanding the Problem and Identifying Information, (2) Constructing Initial Representations, (3) Developing Mathematical Models, (4) Interpreting and Validating Results, and (5) Reflection and Revision. Descriptive statistical analysis was performed by calculating the mean and standard deviation for each indicator and each stage of the model. The mean scores were subsequently interpreted according to the predetermined response categories to describe students' perceptions of the implementation of the mathematical problem-solving model.

The results are first presented by examining students' responses at each stage of the model to provide a detailed understanding of their perceptions throughout the implementation process. Subsequently, the findings are summarized to illustrate the overall response pattern across all stages of the model. This presentation enables the identification of stages that were perceived most positively as well as those requiring further improvement, thereby providing a comprehensive evaluation of the implementation of the mathematical problem-solving model in Integral Calculus. The results are presented in the following tables.

Table 1:
Students' Responses to the Understanding the Problem and Identifying Information Stage

No.	Indicator	Mean	Category
1	Clarity of Learning Objectives and Initial Instructions	4,62	Very High
2	Understanding the Problem Context	4,46	Very High
3	Identification of Known and Required Information	4,61	Very High
4	Engagement in Understanding the Problem	4,62	Very High
	Overall Mean for Stage 1	4,58	Very High

Based on the results presented in Table 1, students' responses to the implementation of the mathematical problem-solving model at Stage 1 (Understanding the Problem and Identifying Information) were classified as very high, with an overall mean score of 4.58. All indicators at this stage achieved mean scores ranging from 4.46 to 4.62, indicating that students responded very positively to the learning activities during the initial phase of the model. The indicators "Clarity of Learning Objectives and Initial Instructions" and "Engagement in Understanding the Problem" obtained the highest mean scores (M = 4.62), suggesting that students clearly understood the learning objectives and actively participated in comprehending the given problems. Furthermore, the indicator "Identification of Given and Required Information" achieved a mean score of 4.61, indicating that students were able to identify relevant information effectively as the basis for developing appropriate problem-solving strategies.

In contrast, "Understanding the Problem Context" received the lowest mean score (M = 4.46), although it remained within the very high category. This finding suggests that understanding the contextual aspects of mathematical problems was relatively more challenging than the other indicators at this stage. Overall, the findings demonstrate that the implementation of the proposed mathematical problem-solving model effectively supported students in developing an initial understanding of mathematical problems by providing clear instructional guidance, facilitating the identification of relevant information, and encouraging active engagement throughout the learning process. The highly positive responses at this stage indicate that students were well prepared to proceed to the next stage, Constructing Initial Representations, which serves as the foundation for the mathematization and mathematical modeling processes.

Table 2:
Descriptive Statistics of Students' Responses at the Constructing Initial Representations Stage

No.	Indicator	Mean	Category
1	Ease of Constructing Initial Representations	4,14	High
2	Identification of Mathematical Variables and Symbols	4,14	High
3	Transformation of Contextual Problems into Mathematical Forms	4,08	High
4	Clarity of Relationships Among Mathematical Representations	4,22	Very High
	Overall Mean for Stage 2	4,14	High

Based on the results presented in Table 2, students' responses to the implementation of the mathematical problem-solving model at Stage 2 (Constructing Initial Representations) were classified as high, with an overall mean score of 4.14. The mean scores for all indicators ranged from 4.08 to 4.22, indicating that students responded positively to the learning activities associated with constructing initial mathematical representations. Among the indicators, "Clarity of Relationships Among Mathematical Representations" obtained the highest mean score (M = 4.22), suggesting that students were able to understand the relationships among multiple forms of mathematical representations used throughout the problem-solving process. Meanwhile, the indicators "Ease of Constructing Initial Representations" and "Identification of Mathematical Variables and Symbols" each achieved a mean score of 4.14, indicating that most students were able to construct initial representations and identify the mathematical variables and symbols required for solving the given problems.

In contrast, "Transformation of Contextual Problems into Mathematical Forms" received the lowest mean score (M = 4.08), although it remained within the high category. This finding suggests that transforming contextual situations into mathematical forms was relatively more challenging than the other activities performed during the initial representation stage. Overall, the findings indicate that the proposed mathematical problem-solving model effectively supported students in constructing initial representations before proceeding to formal mathematical problem-solving. Nevertheless, compared with the other stages of the model, Stage 2 obtained a relatively lower mean score, indicating that students' initial mathematization processes, particularly the transformation of contextual problems into mathematical forms, still require further instructional support in future implementations.

Table 3:
Descriptive Statistics of Students' Responses at the Developing Mathematical Models Stage

No.	Indicator	Mean	Category
1	Selection of Solution Strategies	4,26	Very High
2	Application of Integral Concepts and Procedures	4,28	Very High
3	Systematic Development of Mathematical Models	4,04	Very High
4	Monitoring the Problem-Solving Process	4,22	Very High
5	Active Participation in Discussions and Strategy Explanation	4,30	Very High
	Overall Mean for Stage 3	4,22	Very High

Based on the results presented in Table 3, students' responses to the implementation of the mathematical problem-solving model at Stage 3 (Developing Mathematical Models) were classified as very high, with an overall mean score of 4.22. This finding indicates that students responded very positively to the learning activities associated with developing mathematical models. Overall, the mean scores of the indicators ranged from 4.04 to 4.30, suggesting that all aspects of this stage were positively perceived by the students. Among the indicators, "Active Participation in Discussion and Explaining Solution Strategies" obtained the highest mean score (M = 4.30), indicating that students actively engaged in discussions and were able to communicate their solution strategies effectively throughout the learning process. In addition, "Application of Integral Concepts and Procedures" achieved a mean score of 4.28, while "Selection of Appropriate Solution Strategies" obtained a mean score of 4.26. These findings indicate that students were able to select appropriate solution strategies and apply relevant integral concepts and procedures to solve the given problems.

Meanwhile, "Monitoring the Problem-Solving Process" received a mean score of 4.22, remaining within the very high category. In contrast, "Systematic Development of Mathematical Models" obtained the lowest mean score (M = 4.04), although it was still classified as very high. This finding suggests that organizing mathematical models into a systematic solution process remained relatively more challenging than the other activities at this stage. Overall, the results demonstrate that the proposed mathematical problem-solving model effectively supported students in developing mathematical solutions through appropriate strategy selection, application of integral concepts, continuous monitoring of the solution process, and active participation in discussions. Nevertheless, the relatively lower score for the systematic development of mathematical models indicates that this aspect remains an area for further improvement in future implementations of the model.

Table 4:
Descriptive Statistics of Students' Responses at the Interpreting and Validating Results Stage

No.	Indicator	Mean	Category
1	Interpretation of Results in Context	4,26	Very High
2	Validation of the Accuracy and Reasonableness of Results	4,19	High
3	Evaluation of Solution Strategies and Procedures	4,22	Very High
4	Ease of Interpreting and Validating Results	4,34	Very High
	Overall Mean for Stage 4	4,25	Very High

Based on the results presented in Table 4, students' responses to the implementation of the mathematical problem-solving model at Stage 4 (Interpreting and Validating Results) were classified as very high, with an overall mean score of 4.25. This finding indicates that students responded very positively to the learning activities involving the interpretation of problem-solving results and the validation of the solutions obtained. Overall, the mean scores of the indicators ranged from 4.19 to 4.34, indicating that all aspects of the interpretation and validation stage were perceived very positively by the students. Among the indicators, "Ease of Interpreting and Validating Results" obtained the highest mean score (M = 4.34), suggesting that students perceived the processes of interpreting solution outcomes and validating their answers as clear and manageable within the implemented learning model. Furthermore, "Interpretation of Results in Context" achieved a mean score of 4.26, while "Evaluation of Problem-Solving Strategies and Procedures" obtained a mean score of 4.22. These findings indicate that students were able to relate their solutions to the problem context and critically evaluate the strategies and procedures employed during the problem-solving process.

In contrast, "Validation of the Accuracy and Reasonableness of Results" received the lowest mean score (M = 4.19), although it remained within the very high category. This finding suggests that verifying the mathematical accuracy and contextual reasonableness of the obtained solutions remained relatively more challenging than the other activities at this stage. Overall, the results demonstrate that the proposed mathematical problem-solving model effectively supported students in interpreting solution outcomes, evaluating the strategies employed, and validating the solutions obtained. The highly positive responses at this stage indicate that interpretation and validation were successfully integrated as essential components of the mathematical problem-solving process.

Table 5:
Descriptive Statistics of Students' Responses at the Reflection and Revision Stage

No.	Indicator	Mean	Category
1	Reflection on the Problem-Solving Process	4,36	Very High
2	Identification and Revision of Solution Errors	4,32	Very High
3	Development of Alternative Solution Strategies	4,42	Very High
4	Motivation and Self-Confidence After Reflection	4,53	Very High
	Overall Mean for Stage 5	4,41	Very High

Based on the results presented in Table 5, students' responses to the implementation of the mathematical problem-solving model at Stage 5 (Reflection and Revision) were classified as very high, with an overall mean score of 4.41. This finding indicates that students responded very positively to learning activities focusing on reflection, evaluation of solution outcomes, and revision of the solutions obtained. Overall, the mean scores of the indicators ranged from 4.32 to 4.53, indicating that all aspects of the reflection and revision stage were perceived very positively by the students. Among the indicators, "Motivation and Self-Confidence After Reflection" obtained the highest mean score (M = 4.53), suggesting that the reflection process not only enabled students to evaluate their problem-solving outcomes but also enhanced their learning motivation and self-confidence in solving mathematical problems. In addition, "Development of Alternative Solution Strategies" achieved a mean score of 4.42, indicating that students were encouraged to explore and develop alternative solution strategies as part of the reflective process.

Meanwhile, "Reflection on the Problem-Solving Process" obtained a mean score of 4.36, whereas "Correction of Errors and Solution Revision" received the lowest mean score (M = 4.32). Although these indicators obtained the lowest scores at this stage, they remained within the very high category. These findings indicate that students were able to reflect on their problem-solving processes and revise errors identified during the solution process. Overall, the results demonstrate that the proposed mathematical problem-solving model effectively facilitated systematic reflection on both the problem-solving process and its outcomes. The highly positive responses at this stage suggest that reflection and revision provided meaningful learning experiences that contributed not only to improving the quality of students' solutions but also to strengthening their confidence in solving similar mathematical problems in future learning situations. A summary of students' responses across all five stages of the model is presented in the following table.

Table 6:
Overall Students' Responses to the Implementation of the Mathematical Problem-Solving Model

No.	**Model Stage**	Mean	Category
1	Understanding the Problem and Identifying Information	4,58	Very High
2	Constructing Initial Representations	4,14	High
3	Developing Mathematical Models	4,22	Very High
4	Interpreting and Validating Results	4,25	Very High
5	Reflection and Revision	4,41	Very High
	Overall Mean Across All Stages	4,42	Very High

Based on the results presented in Table 6, students demonstrated an overall very high level of response to the implementation of the mathematical problem-solving model in the Integral Calculus course, with an overall mean score of 4.42. This finding indicates that all stages of the proposed model were well accepted and positively perceived by the students throughout the learning process. Among the five stages, Stage 1 (Understanding the Problem and Identifying Information) achieved the highest mean score (M = 4.58), suggesting that students experienced little difficulty in understanding the learning objectives, identifying the given and required information, and comprehending the problem context before initiating the solution process. In contrast, Stage 2 (Constructing Initial Representations) obtained the lowest mean score (M = 4.14). Although this stage was still classified within the high category, the result indicates that transforming contextual problems into mathematical representations remained relatively more challenging than the other stages.

Stage 3 (Developing Mathematical Models) achieved a mean score of 4.22, whereas Stage 4 (Interpreting and Validating Results) obtained a mean score of 4.25. These findings indicate that students responded positively to activities involving strategy selection, the application of integral concepts and procedures, and the interpretation and validation of mathematical solutions. Furthermore, Stage 5 (Reflection and Revision) obtained a mean score of 4.41, indicating that students perceived reflective activities, evaluation of the problem-solving process, and solution revision very positively. Overall, the findings reveal a consistently positive response across all five stages of the model, with mean scores ranging from 4.14 to 4.58. Although slight variations were observed among the stages, the differences were relatively small, suggesting that the model consistently supported students throughout the mathematical problem-solving process, from understanding problems and constructing mathematical representations to developing mathematical models, interpreting results, and engaging in reflection and revision.

Overall, these findings indicate that the implementation of the proposed mathematical problem-solving model was positively received across all stages of learning. Nevertheless, the Constructing Initial Representations stage received the lowest mean score, identifying initial mathematization as the stage requiring the greatest instructional attention in future implementations of the model within Integral Calculus. Conversely, the highest response observed at the Understanding the Problem and Identifying Information stage suggests that the model successfully established students' initial conceptual understanding, providing a strong foundation for subsequent mathematical problem-solving activities.

DISCUSSION

The findings revealed that the implementation of the mathematical problem-solving model in the Integral Calculus course received a very positive overall response, with an overall mean score of 4.42. This result indicates that students responded favorably to all stages of the problem-solving process, ranging from understanding the problem to reflection and revision. However, students' responses did not follow a uniform pattern. The highest response was observed at the Understanding the Problem and Identifying Information stage, whereas the Constructing Initial Representations stage received the lowest response before increasing again during the Developing Mathematical Models, Interpreting and Validating Results, and Reflection and Revision stages. This pattern suggests that students' responses reflected the varying cognitive demands associated with different stages of mathematical problem solving rather than merely indicating their acceptance of the proposed model. Unlike previous studies that primarily evaluated the effectiveness of problem-solving models through improvements in learning outcomes or problem-solving performance, the present study demonstrates that students' perceptions vary systematically across different cognitive stages of the problem-solving process. This finding highlights the importance of evaluating the implementation of problem-solving models from a process-oriented perspective rather than relying solely on outcome-based measures.

The highest response at the Understanding the Problem and Identifying Information stage indicates that students experienced relatively little difficulty in comprehending the problem context, identifying the given and required information, and establishing appropriate solution goals. This finding reinforces Polya's (1957) proposition that understanding the problem constitutes the foundation of the entire problem-solving process. Without an adequate understanding of the problem situation, subsequent strategy selection is unlikely to be effective. From a cognitive perspective, this stage primarily involves activating prior knowledge, organizing relevant information, and constructing an initial mental representation of the problem without requiring formal mathematical transformation. Consequently, the cognitive demands remain relatively lower than those encountered during later stages. This interpretation is supported by recent studies demonstrating that successful mathematical problem solving depends not only on obtaining correct answers but also on the quality of students' conceptual understanding, representation, communication, and reflective thinking throughout the solution process. Students possessing more comprehensive conceptions of mathematical problem solving consistently demonstrate higher performance across these dimensions (Clark, 2020; Dixon & Moore, 1997; Novitasari et al., 2020). Therefore, the strong response observed at this stage suggests that establishing an initial conceptual understanding provides an essential foundation for the successful completion of subsequent problem-solving activities.

The most notable finding of this study is that the Constructing Initial Representations stage obtained the lowest mean score among all stages. Although students' responses remained within the high category, this result suggests that mathematization constitutes the most cognitively demanding phase of the problem-solving process. More specifically, the indicator related to transforming contextual problems into mathematical forms received the lowest score among all indicators. This finding indicates that students experienced relatively few difficulties in understanding mathematical problems; however, greater challenges emerged when they were required to transform contextual situations into mathematical representations that could serve as the basis for further problem solving. This result represents an important contribution of the present study because it identifies mathematization as the principal cognitive bottleneck within the implementation of the mathematical problem-solving model.

This finding is consistent with mathematization theory, which conceptualizes mathematization as a translation process between real-world situations and formal mathematical representations (Arifin et al., 2021; Jupri et al., 2021; Mansour et al., 2021). During this phase, students must identify relevant variables, formulate assumptions, select appropriate representations, and establish mathematical relationships capable of representing the original problem situation. Consequently, mathematization requires substantially greater cognitive resources than merely understanding the problem context. Furthermore, Duval (2006) argued that one of the principal sources of mathematical learning difficulties lies in students' ability to transform information across multiple representational systems, such as converting verbal descriptions into symbolic, graphical, or algebraic representations. Therefore, the relatively lower response observed during the initial representation stage should not be interpreted simply as evidence of students' weaknesses but rather as a natural consequence of the cognitive complexity inherent in mathematical problem solving.

Successful mathematical problem solving is strongly influenced by students' ability to construct, coordinate, and transition among multiple mathematical representations. Students capable of integrating verbal, visual, symbolic, and graphical representations generally demonstrate deeper conceptual understanding than those relying on a single representational form. Conversely, limited representational flexibility often constrains students' ability to develop mathematical models that appropriately capture the contextual characteristics of a problem. These findings further support the present results by indicating that the Constructing Initial Representations stage represents the critical phase requiring the greatest instructional attention during the implementation of mathematical problem-solving models (Krawec, 2012; Novitasari et al., 2020; Swidan & Yerushalmy, 2016; Zazkis, 2016). Consequently, strengthening students' representational competence should be considered a priority for improving the effectiveness of mathematical problem-solving instruction.

Interestingly, students' responses increased substantially after successful construction of mathematical representations. Higher responses were observed during the Developing Mathematical Models, Interpreting and Validating Results, and Reflection and Revision stages. This pattern indicates that well-established mathematical representations served as cognitive foundations enabling students to select appropriate solution strategies, apply integral procedures, interpret mathematical results, and evaluate the validity of their solutions. In other words, successful mathematization appears to function as a prerequisite for the effectiveness of subsequent stages of mathematical problem solving. This interpretation is consistent with Schoenfeld's (1985) view that mathematical problem solving is inherently dynamic, with the quality of initial representations exerting a substantial influence on subsequent strategy selection, monitoring, and evaluation processes.

The Reflection and Revision stage received the second-highest response after the problem understanding stage. This finding indicates that students positively perceived activities involving reflection on their thinking processes, verification of obtained solutions, correction of errors, and exploration of alternative solution strategies. According to Polya (1957), the *looking back* phase extends beyond merely verifying the final answer; rather, it involves evaluating the effectiveness of solution strategies and constructing transferable knowledge applicable to future mathematical problems. Recent studies have similarly demonstrated that reflective thinking is significantly associated with mathematical problem-solving performance (Artzt & Armour-Thomas, 1999; Jiang & Jia-qing, 2024; Pratiwi et al., 2019). Students who perceive problem solving as a process of exploration and reflection consistently demonstrate higher performance than those focusing exclusively on obtaining correct answers. These findings suggest that systematic reflection not only improves solution quality but also strengthens students' confidence and adaptive problem-solving abilities.

Taken together, the findings reveal a progressive cognitive pattern throughout the mathematical problem-solving process. Students' responses were highest during problem understanding, declined during mathematization, and subsequently increased throughout mathematical model development, interpretation, validation, and reflection. This pattern indicates that cognitive complexity does not increase linearly but instead reaches its highest point when students are required to transform contextual problems into formal mathematical representations (Joklitschke et al., 2022; Purnomo et al., 2020, 2022; Purnomo, Suparman, & Kadarwati, 2020; Rott et al., 2021). Once appropriate mathematical representations have been established, the cognitive demands appear to decrease because students already possess an organized mathematical framework that facilitates procedural execution, interpretation, validation, and reflection. This progressive cognitive pattern represents an important theoretical contribution by demonstrating that students' responses are closely associated with the cognitive complexity of each stage of mathematical problem solving rather than being uniformly distributed across the entire problem-solving process.

The primary contribution of this study lies in providing empirical evidence that evaluating students' responses at each stage of a mathematical problem-solving model offers deeper insights into the cognitive characteristics underlying problem-solving processes than evaluations based solely on learning outcomes. The findings identify mathematization as the principal cognitive bottleneck within Integral Calculus problem solving and emphasize the importance of strengthening representational competence before students proceed to formal mathematical processing. Theoretically, this study extends current understanding of stage-based mathematical problem-solving processes by demonstrating that students' responses vary according to the cognitive demands of each stage. Methodologically, it introduces a process-oriented approach for evaluating the implementation of mathematical problem-solving models through stage-specific student responses. Practically, the findings suggest that future instructional interventions should prioritize activities that enhance mathematization, mathematical representation, mathematical modeling, and representational translation to improve students' overall mathematical problem-solving performance in Integral Calculus.

CONCLUSION

The implementation of the mathematization- and mathematical modeling-based problem-solving model received a very positive response from students in the Integral Calculus course (M = 4.42). The highest response was observed at the Understanding the Problem and Identifying Information stage, whereas the Constructing Initial Representations stage received the lowest response, identifying mathematization as the most challenging phase of the problem-solving process. Overall, the findings demonstrate that the model effectively supports students' systematic engagement in mathematical problem solving. Furthermore, the study provides empirical evidence that stage-based analysis of students' responses is valuable for evaluating model implementation and identifying stages requiring further instructional support. Future research should investigate the relationship between students' responses, problem-solving performance, and mathematization competence, as well as examine the model across different Calculus topics and other mathematics learning contexts.

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