Mixed-Mode Fracture Analysis Based on Elastic-Plastic Fracture Mechanics

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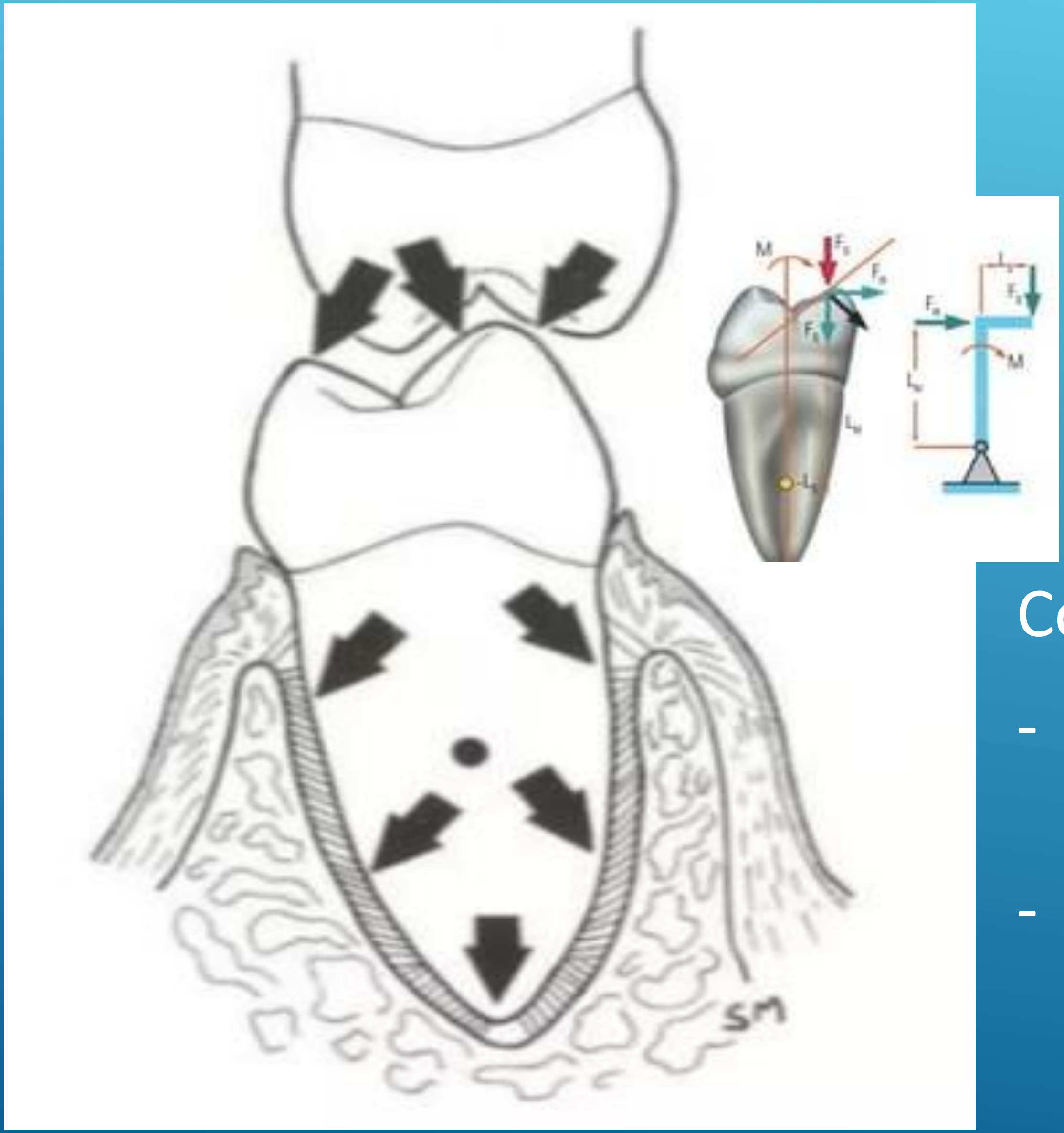
Introduction

Structures or body often experience complex loads, such as plane stress and in-plane and out-of-plane shear stresses. The resistance to crack propagation in fracture toughness studies is often analyzed by assuming that cracks occur in Mode I fracture mechanics

In practice, multiaxial stress or angled cracks can cause mixed-mode fractures in structural materials and components

Mixed mode fractures are frequently the result of complex loading conditions, which have been studied in great detail utilizing a variety of combinations, including mixed mode I/II, mixed mode II/III, mixed mode I/III, and mixed mode I/II/III. From the initial crack, mixed-mode load produces a complex crack that tilts and twists, resulting in a complex fracture surface

For instance: force in teeth

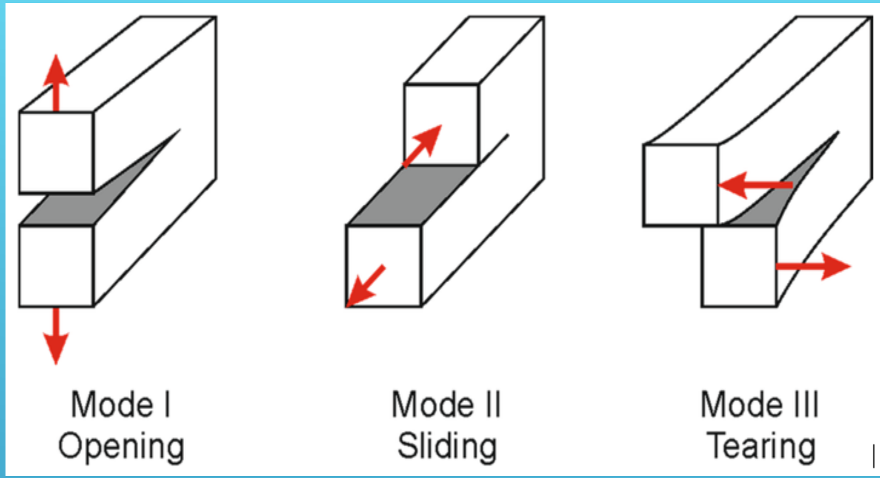


In their use such as chewing and speaking, dentures experience different loading due to its complex geometry and varying three-dimensional stresses.

Complex Stress:

- When force is applied over a body, complex of multiple stresses is produced
- They may be a combination of tensile, shear or compressive stress.

Due to the fairly complex loading situation, the fracture mechanics of the material is no longer on single-mode fractures but focuses more on various mixed-mode fractures



The complex stress that occurs in fractured materials is classified into three stress orientations:

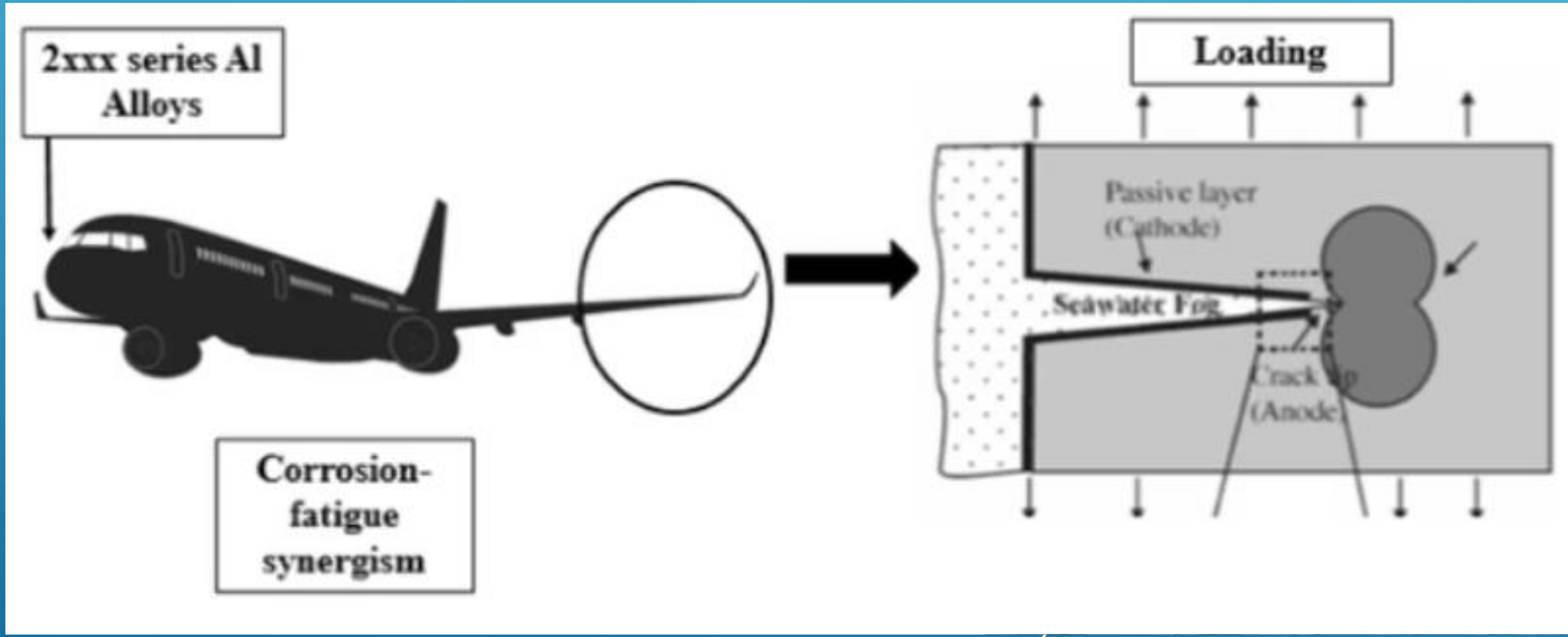
(i) a tensile stress normal to the local plane of the crack surface (Mode I - opening mode);

(ii) parallel to the surface but perpendicular to the crack face (Mode II - shear mode); and

(iii) shear stress acting parallel to the surface and crack face (Mode III - tearing mode).

An example of a case that often occurs is a thin sheet of aircraft aluminum (Al) alloy which often shows stable tear propagation before experiencing total fracture. The stability of this fracture propagation should encourage researchers to determine the tear mechanism in mixed load modes.

The characteristics of the fractures that occur can be used to determine the right research method so that a comprehensive design and analysis can be found to prevent fatal damage to aircraft components.

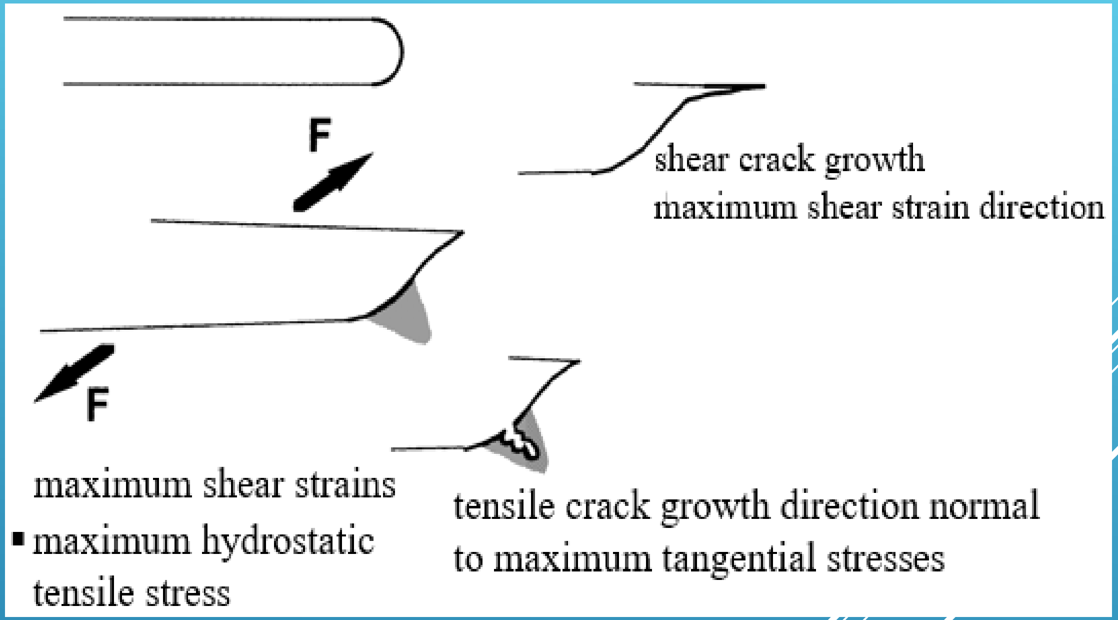


Mixed-Mode Crack Initiation and Propagation

The investigation of mixed-mode I/II in elastic-plastic materials has involved numerous experimental studies and finite element simulations.

- studies were conducted under mixed mode loading, and the damage region surrounding the crack tip was observed after the fracture process occurred.

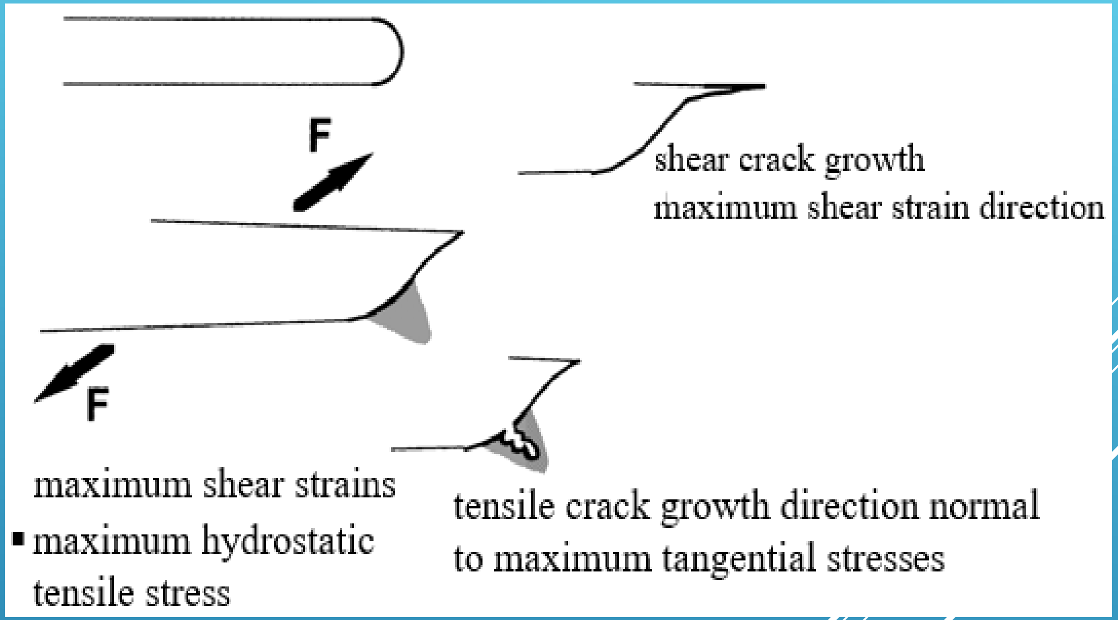
- the damage in the fracture zone was non-uniform, indicating the dominance of tensile stress.



Crack tip deformation and crack propagation due to mixed mode loading on ductile materials

Mixed-Mode Crack Initiation and Propagation

- The zone at the crack tip is the focus of study in determining the fracture mechanism of the material.
- Mode I is marked by the appearance of a tensile fracture mechanism in the blunt area.
- Mode II is marked by the presence of a shear mechanism in the sharp part of the deformed crack tip



Numerous elements influence the prevailing fracture mode, such as the loading combination, work hardening behavior, and microstructure (the quantity and presence of inclusions, porosity, etc.)

Mixed mode fracture toughness is influenced by several factors, including the crack propagation mode, and the choice of fracture parameters.

Fracture criteria in the framework of LEFM (Linier Elastic Fracture Mechanics)

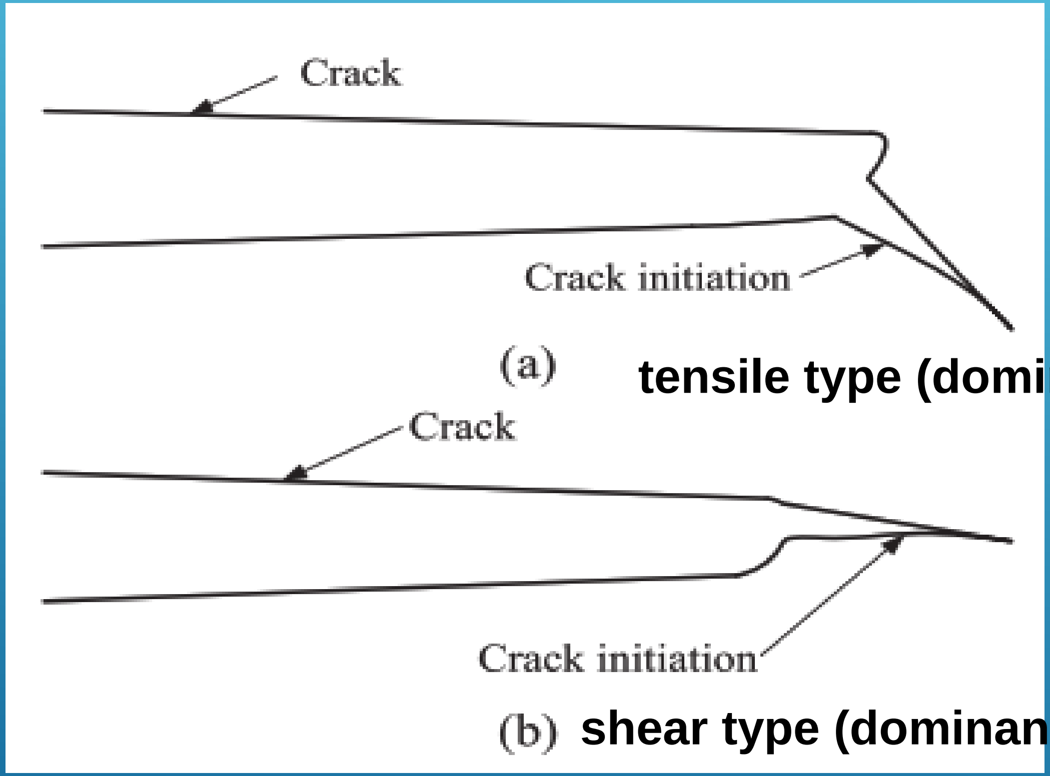
- fracture criteria that can be used to predict the initiation and propagation of the crack under mixed-mode fracture:
  - the maximum tangential stress at the crack tip under mixed-mode loading
  - a minimum energy strain density (MSED) criteria to predict the direction of fracture propagation.
  - the Maximum Energy Release Rate (MERR) criteria.

Fracture criteria in the framework of EPFM (Elastic-Plastic Fracture Mechanics)

- The J-Integral: stress and strain fields at the crack tip of nonlinear elastic materials
- crack tip opening displacement (CTOD) as a measure of fracture toughness in the presence of plasticity

Recently, the development of EPFM has led to the progressive establishment of mixed-mode fracture criteria for ductile or elastoplastic materials. For instance, Ma et al. developed CTOD-based criteria for predicting the onset and direction of crack growth under mixed-mode loading for ductile fracture behavior.

two load types (for Al alloys), i.e., tensile and shear, are the initial loads that initiate cracks during mixed mode I/II fractures:

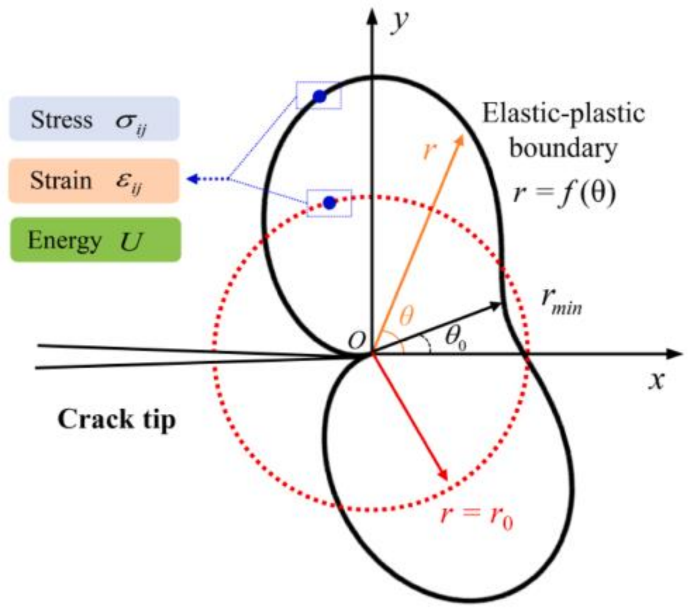


tensile type (dominance in Mode I)

shear type (dominance in Mode II)

core area of the crack tip

- Plastic yield, which then undergoes local elastic-plastic deformation at the crack tip, is very important to study to determine the fracture behavior of the material.
- The existence of a plastic zone at the crack tip can prevent the formation of new cracks and inhibit fracture propagation.
- The crack initiation angle (see Figure) is produced from a mechanical process in a circular area with a constant radius r_0.



Schematics diagram showing the core area of the crack tip

Many fracture criteria have also been developed due to the presence of an elastic-plastic core zone:

- the minimum spacing needed in the plastic zone → when the octahedral shear stress component reaches a threshold value, crack initiation in the plastic zone continues.
- the first fracture starts to spread when the aggregate strain energy in the area enclosed by the boundary line hits a threshold value.
- the W criterion, which is based on the idea that the minimum radius of the elastic-plastic core area surrounding the crack tip can be used to calculate the fracture initiation angle. In the framework of elastic-plastic materials
- R-criterion concerning the shapes and sizes of the crack tip plastic center region in mixed-mode I/II fracture criteria.
- minimum plastic zone radius (MPZR) criterion, which was developed by referring to the Von Mises yield criterion

Mixed-Mode Fracture Parameters

The key factors in fracture mechanics parameters:

- ❑ Elastic strain energy release rate
- ❑ Stress Intensity Factor (SIF)

} **linier elastic fracture mechanics (LEFM)**

- ❑ J-integral
- ❑ Crack Tip Opening Displacement

} **Elastic-plastic fracture mechanics (EPFM)**

- ➢ LEFM → does not consider plastic yielding at or near the crack tip during crack propagation and is related to brittle materials
- ➢ EPFM → consider plastic yielding at or near the crack tip was appear during crack propagation

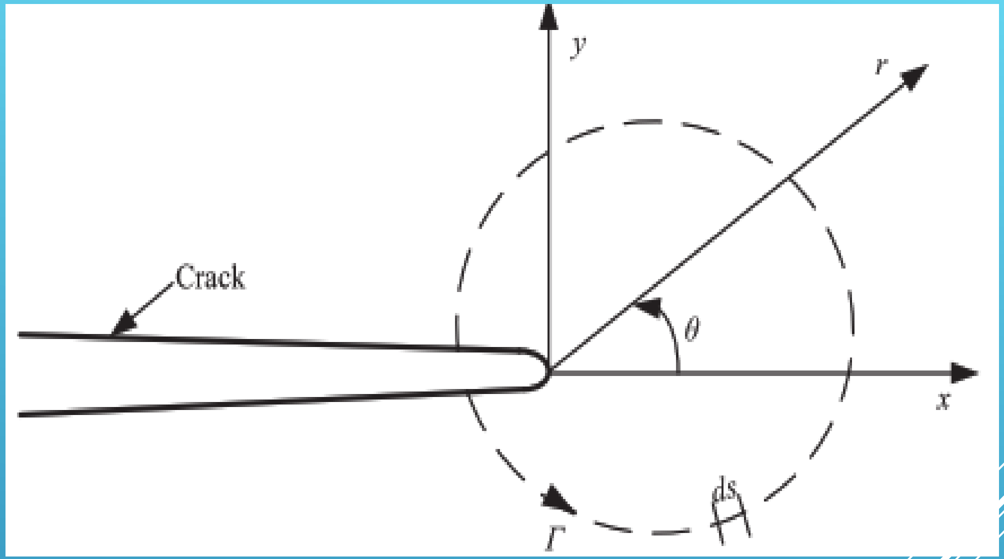
J-integral parameter : to characterize elastic-plastic materials in terms of the properties of the stress and strain fields at the crack tip.

- for every unit area of the fracture surface, the J-integral may show how quickly the total potential energy in an elastic-plastic material changes.
- The energy released per unit fracture surface area is thus represented by the J-integral, a line integral (path-independent) around the crack tip.
- So, it can be written as:

$$J = \int_{\Gamma} \left(w dy - Ti \frac{\partial ui}{\partial x} ds \right) \quad (4)$$

where w, T_i, u_i, ds, and Γ are the strain energy density, the vector traction components, displacement vector components, increment length of the contour, and an arbitrary curvature surrounding the crack, respectively.

Figure. J-integrals on arbitrary integration path under mixed-mode loads of mode I and mode II



Global coordinate system and path-independent integral related to J-integral in its entirety, where x and y are a point's Cartesian coordinates

For materials with stress and plastic strain fields at the crack tip, the power-law relationship can be written as follows:

$$\sigma_{ij} = a_0 + \left(\frac{EJ}{\alpha\sigma_0^2 I_n r}\right)^{1/(n+1)} \widetilde{\sigma_{ij}}\,(n,\theta) \quad (5)$$

$$\varepsilon_{ij} = \frac{\alpha\sigma_0}{E}\left(\frac{EJ}{\alpha\sigma_0^2 I_n r}\right)^{1/(n+1)} \widetilde{\sigma_{ij}}\,(n,\theta) \quad (6)$$

where n is the strain-hardening exponent. The tensors of stress and strain are denoted by σ_{ij} and ε_{ij}, respectively. Typically, the yield stress and the reference stress, σ_0, are equal. The parameters α and E are the dimensionless constant and Young's modulus, respectively.

The CTOD is an additional metric for describing the fracture behavior of ductile materials.

- The CTOD on stress triaxiality, as illustrated in Fig., is the average displacement across two opposed locations or distance at the 90° intercept location between a crack tip's opposing faces, i.e., the upper and bottom of crack edges [88].
- the CTOD represents the fracture tip displacement vectors.
- Many tests on ferritic steel with the goal of examining the CTOD properties under mixed mode I/II loading show that the J-CTOD was found in the HRR (Hutchinson-Rice-Rosengren) singularity [9].
- Therefore, it can be written as:

$$CTOD = (\alpha\varepsilon_0)^{1/n} \frac{J}{\sigma_0} D_N \qquad (7)$$

where D_N is a function of mode mixity and $\varepsilon_0 = \sigma_0/E$. It is evident that $(\alpha\varepsilon_0)1/n \approx 1$ for materials with high strain hardening exponent values, thus CTOD's reliance on $(\alpha\varepsilon_0)^{1/n}$ is minimal.

In LEFM and EPFM-based analysis, CTOD is an additional parameter that can be utilized to assess the direction of fracture initiation and propagation.

- According to this criterion, cracks propagate when CTOD approaches a certain threshold.
- The crack growth direction is perpendicular to the largest CTOD (LEFM-based analysis).
- However, under EPFM-based analysis conditions, the competition between the components of the opening mode (mode I) and in-plane shear (mode II) can be used to estimate the crack type under mixed mode I/II loading.

In LEFM-based analysis, i.e., in brittle materials, the CTOD criterion is considered to be the same as the maximum energy release rate (MERR) and maximum confining stress (MCS) criteria.

- Based on the CTOD criteria, the prediction of fracture type in ductile materials shows significant agreement with the experimental results.
  - the CTOD criterion can be used precisely in predicting how ductile fracture will propagate.

Calculation of J-Integral

The J-integral concept was initially put forth using the energy technique, which is organized as an integral value that solely depends on the path's endpoints and not on the precise path that was followed to get there. Therefore,

- the integral value is equal to the potential energy loss per unit fracture increment in elastic materials. J-Integral could be considered a measure of the stress and strain at the notch and crack tip because it does not depend on the route. Therefore,
  - it can be interpreted as J-integral as a stress intensity parameter in addition to an energy parameter.

This concept was developed and widely used, especially in fracture toughness calculations. In fracture mechanisms, J-integral is the sum of the plastic component (J_{pl}) and elastic component (J_e), therefore it can be expressed as follows:

$$J = J_{el} + J_{pl} \quad (11)$$

The elastic part is estimated as follows:

$$J_{el} = G = \frac{K_I^2}{E'} + \frac{K_{II}^2}{E'} + \frac{K_{III}^2}{2\mu} \quad (12)$$

The plastic component of J-integral can be written as follows:

$$J_{pl} = \eta_{pl} \frac{U_{pl}}{B(W-a)} \quad (13)$$

where G is the elastic strain energy release rate, $E' = E$ for plane stress, and $E' = E/(1-\nu^2)$ for plane strain. The parameters E and μ are Young's modulus and shear modulus of materials, respectively. Mode I, Mode II, and Mode III stress intensity factors are represented by the K_I, K_{II}, and K_{III}, respectively.

where the U_p and B are the plastic work and the specimen thickness, respectively. The parameter a is the initial crack length, and W is the specimen width.

The amount of plastic work U_{pl} is determined by removing the elastic component ($1/2 V_{el}.P$) and integrating the load (*P*)-point displacement (*V*) curve in Fig. 6.

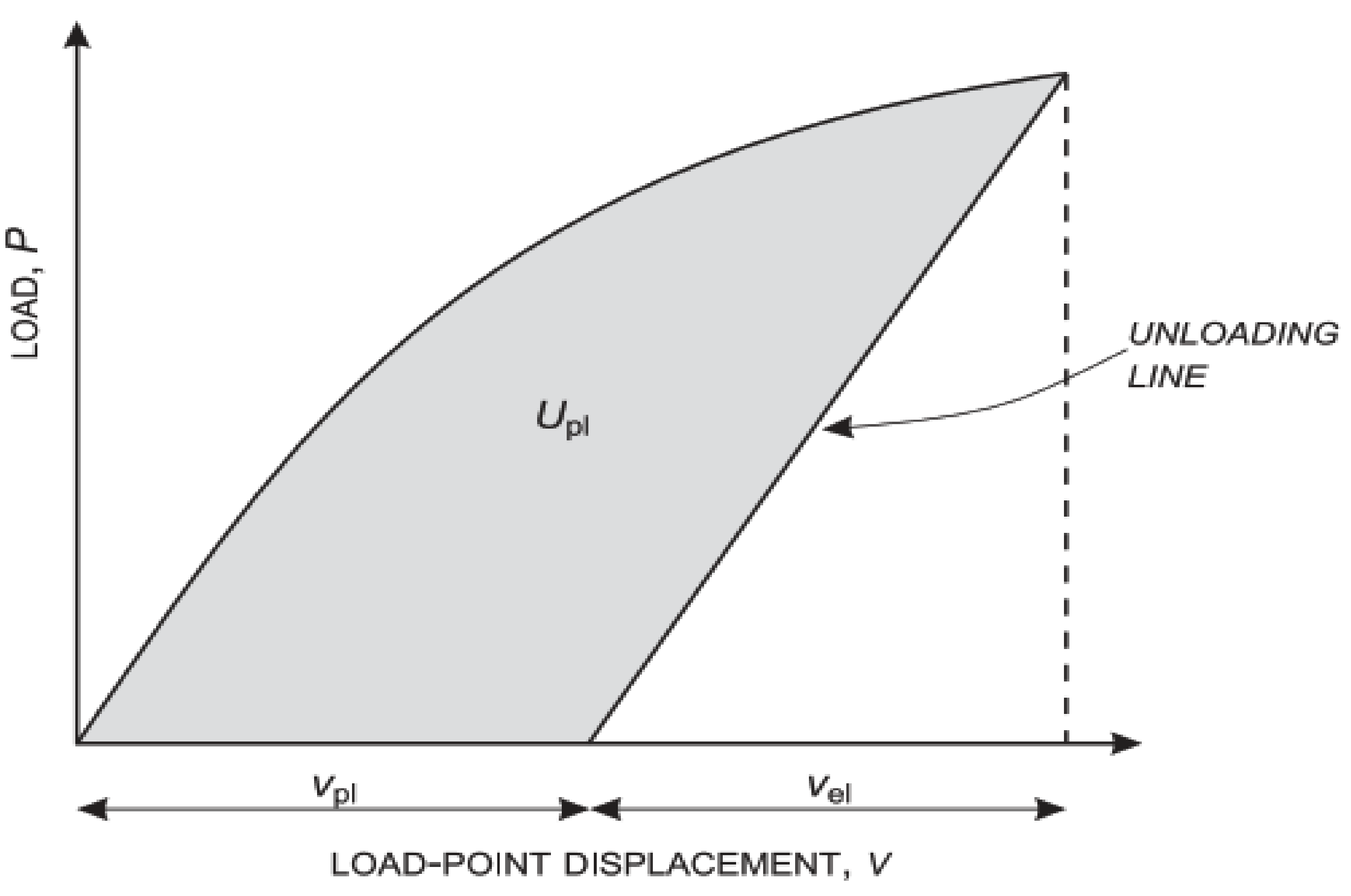
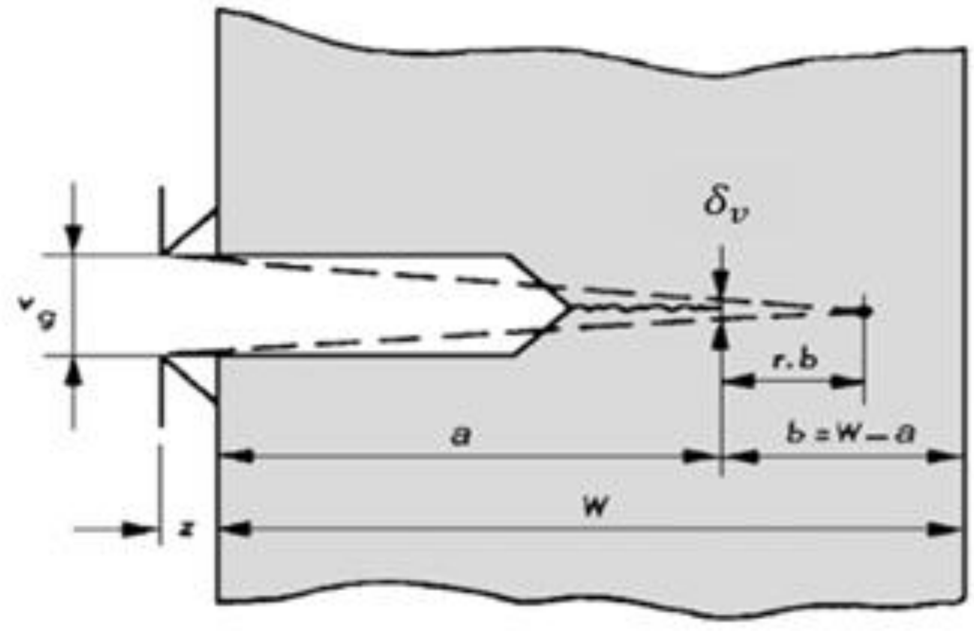


Figure 6. The criteria for plastic work is the area under the P-V curve

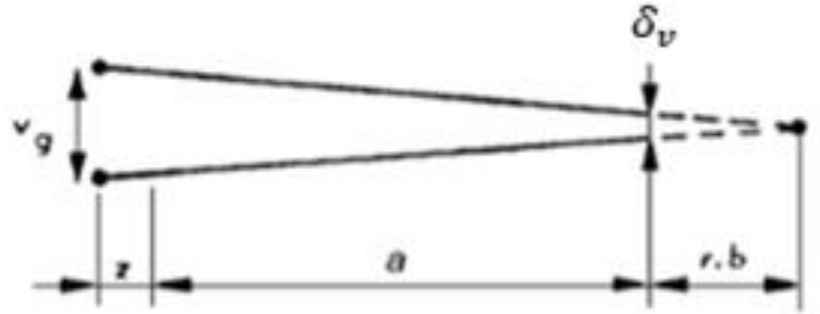
To ensure the validity of the J-integral yield formula:

- the initial crack length *a* (i.e., notch plus initial crack) should be greater than 0.5 W for both the CT and single-edge notched bend (SENB) specimens.
- The ideal value of *a* is 0.6 W, although the longest fracture is 0.75 W

Measurement of CTOD



(a)



(b)

The CTOD (δ_v) varies along the crack front, therefore, it is not possible to measure it directly even using a clip gauge. Alternatively, the COD (v_g) at or near the surface of the specimen is measured using a clip gauge. The ligament b is thus thought to function as a plastic hinge. This suggests that the ligament has a rotation point at a distance r·b.

Fig. (a) shows a clipped gauge attached to the edge of a knife that can be attached to the surface of the specimen.

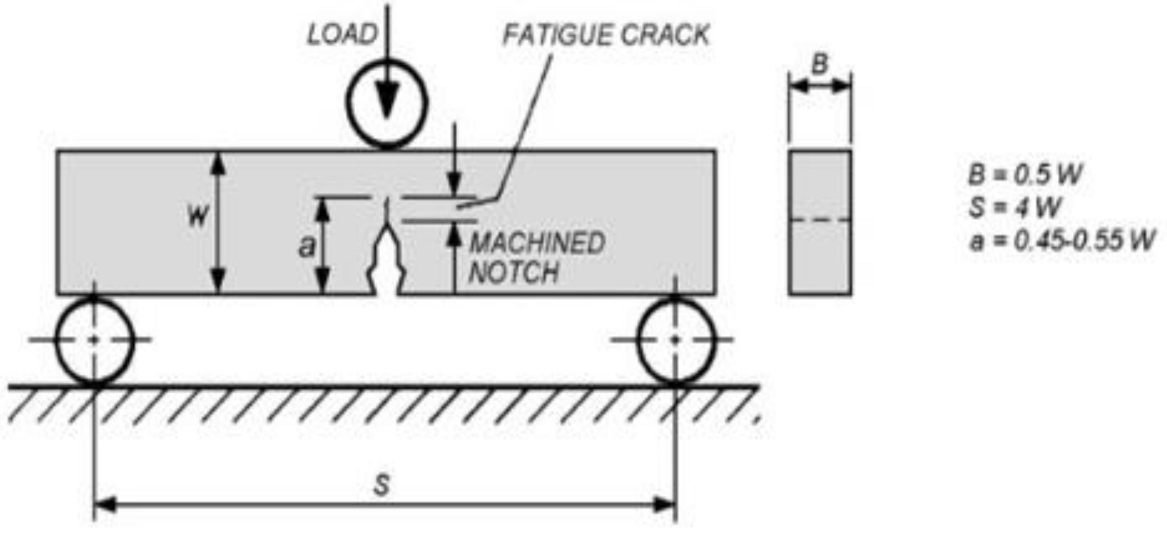
Fig. (b) shows δ_t which can be expressed by

$$\delta_v = \frac{r.b}{r.b+a+z} v_g$$

These parameters v_g can be divided into two parts, i.e. the elastic part v_{el} and the plastic part v_{pl}.

$$\delta_v = \delta_{el} + \delta_{pl}$$

$$= \frac{K_I^2}{E\sigma_{ys}}\left(\frac{1-v^2}{2}\right) + \frac{r.b}{r.b+a+z}\,v_{pl}$$



where the K_I value is calculated based on the standard formula for SENB and CT specimens in the test as illustrated in Fig. 8

For SENB specimen:

$$K_I = \frac{LOAD.S}{BW^{\frac{3}{2}}} \cdot f\left(\frac{a}{W}\right) \tag{21}$$

where

$$f\left(\frac{a}{W}\right) = \frac{3\left(\frac{a}{W}\right)^{\frac{1}{2}}\left[1.99-\frac{a}{W}\left(1-\frac{a}{W}\right)\left\{2.15-3.93\left(\frac{a}{W}\right)+2.7\left(\frac{a}{W}\right)^{2}\right\}\right]}{2\left(1+2\frac{a}{W}\right)\left(1-\frac{a}{W}\right)^{\frac{3}{2}}} \tag{22}$$

For CT specimen:

$$K_I = \frac{LOAD}{BW^{\frac{1}{2}}} \cdot f\left(\frac{a}{W}\right) \tag{23}$$

where

$$f\left(\frac{a}{W}\right) = \frac{\left(2-\frac{a}{W}\right)\left[0.9+4.6\left(\frac{a}{W}\right)-13.3\left(\frac{a}{W}\right)^{2}+14.7\left(\frac{a}{W}\right)^{3}-5.6\left(\frac{a}{W}\right)^{4}\right]}{\left(1-\frac{a}{W}\right)^{\frac{3}{2}}} \tag{24}$$

CTOD measurement is simpler than determining the J-integral under the same fracture conditions

Under these conditions, they divided the CTOD into two components, i.e., the open component and the mixed mode I/II shear displacement. Therefore, the CTOD can be written as:

$$\delta_v = \sqrt{\delta_I^2 + \delta_{II}^2}$$

where δ_I and δ_{II} are the opening and shear displacement

Through optical microscopy investigations, the relative displacement of the gratings on the specimen surface can be used to estimate the values δ_I and δ_{II}. A novel methodology based on the digital image correlation (DIC) technique can be used to observe the crack tip condition, including assessing mixed-mode CTOD.

Conclusion

- Analysis of fracture mechanics and fracture toughness of elastic-plastic materials is most appropriately approached by J-integral and CTOD.
- the fracture toughness of mixed mode I/II increases by the dominance of mode II compared to mode I in the fracture process. Conversely, with the dominance of mode I, the fracture toughness of mixed mode I/II is increased.

Thank You