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Mapping Patterns in Analogical Reasoning: A Qualitative Study of Reflective Middle School Mathematics Students

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Abstract

Analogical reasoning enhances mathematical problem-solving skills in middle school students by enabling knowledge transfer across different contexts. This study investigates how reflective middle school students map structural similarities between source and target mathematical problems and identifies patterns in their analogical reasoning. Through a qualitative case study, two Grade 7 students with reflective cognitive styles calculated the area of a rectangle (source problem) and a parallelogram (target problem). Their problem-solving processes were analyzed using the Structuring, Applying, and Verifying (S-A-V) framework to examine how they identified and mapped structural elements. Findings show that both participants effectively mapped structural elements from the rectangle to the parallelogram, integrating mapping throughout all problem-solving stages. Key patterns include deep structural understanding, high metacognitive engagement, distinct modality preferences (verbal-linguistic and visual-spatial), adaptability in strategy transfer, and rigorous verification practices. These results suggest that reflective cognitive styles significantly enhance analogical reasoning in mathematical problem-solving. Instructional strategies promoting analogical reasoning, structural understanding, metacognitive training, diverse learning styles, and thorough verification can substantially improve mathematical proficiency. However, the limited sample size and focus on reflective styles constrain generalizability, indicating the need for future research with larger, diverse samples and consideration of contextual factors.

Keywords: mapping, analogical reasoning, qualitative study, reflective middle school

INTRODUCTION

Analogical reasoning is a fundamental cognitive process that enables individuals to understand unfamiliar situations by relating them to known concepts [1]. In mathematics education, this process facilitates learning by allowing students to transfer prior knowledge to new problem-solving contexts [2]. Teaching strategies such as bridging analogies leverage analogical reasoning to enhance understanding and promote cognitive development [3]. Among the stages of analogical reasoning—structuring, mapping, applying, and verifying—the **mapping process** is critical. Mapping involves aligning relational structures between a familiar source domain and a new target domain, enabling the identification of correspondences and the inference of unknown elements [4]. This process is essential for effective knowledge transfer and problem-solving in mathematics. Despite its importance, there is a notable gap in research focusing specifically on the mapping process within analogical reasoning among middle school students, particularly those with a reflective cognitive style. **Reflective cognitive style** refers to an individual's tendency to process information carefully and analytically before making decisions [5]. Students with this cognitive style may exhibit unique patterns in mapping due to their propensity for in-depth analysis [6]. By [3] emphasize the need for a deeper understanding of how students develop and apply mapping skills in analogical reasoning. Similarly, highlight that while mapping is crucial for knowledge transfer, its application in mathematical contexts at the secondary education level remains underexplored [7]. Addressing this gap could inform the development of more effective teaching strategies in mathematics.

Purpose of Study

This study aims to explore the mapping patterns in analogical reasoning among reflective middle school mathematics students. Specifically, it seeks to answer the following research questions:

1. How do reflective middle school students perform the mapping process between source and target problems in mathematical analogical reasoning?
2. What patterns emerge during this mapping process?

By investigating these questions, the research intends to provide insights into the cognitive processes of reflective students during mathematical problem-solving. The findings are expected to contribute to the development of instructional methods that enhance analogical reasoning skills.

Significance of Study

Understanding the mapping patterns in analogical reasoning among reflective students has significant implications for mathematics education. Middle school represents a critical stage in the development of abstract thinking abilities [8]. Insights into how reflective students approach mapping can inform educators on tailoring instruction to support deep cognitive processing and improve problem-solving skills.

METHODS

Research Design

This study employed a qualitative approach with a **case study design** to explore the mapping patterns in analogical reasoning among reflective middle school mathematics students. A case study design allows for an in- depth examination of complex phenomena within their real-life context [9], making it suitable for analysing students' cognitive processes during mathematical problem-solving.

Participants

Participants were selected using a sequential purposive sampling strategy [10], focusing on Grade 7 students (aged 12–13 years) from Chung Chung Christian School in Surabaya, East Java, Indonesia. The selection process involved five steps, as illustrated in Figure 1. In the first step, a mathematics ability test was administered to all Grade 7 students to assess their proficiency in key mathematical concepts relevant to the curriculum, such as fractions, decimals, and basic algebra. The test comprised two multiple-choice and three open-ended questions, which were validated by mathematics education experts to ensure content validity. Students were scored out of 100%. In the second step, students scoring between 60% and 80% were classified as having medium mathematical ability. Focusing on students with medium ability was intended to minimize the influence of extreme proficiency levels on the mapping process, thereby providing a balanced perspective. The third step involved administering Kagan (1965) the Matching Familiar Figures Test (MFFT) [11] to assess the cognitive styles of the selected students. The MFFT measures response times and accuracy to distinguish between reflective and impulsive cognitive styles. Longer response times combined with higher accuracy indicated a reflective cognitive style. In the fourth step, students who demonstrated strong reflective characteristics based on the MFFT were identified. A total of five students met the criteria for a reflective cognitive style. Finally, in the fifth step, the highest-scoring male and the highest-scoring female from the group of reflective students were selected as the main participants. This selection aimed to explore the mapping patterns in analogical reasoning among students who exemplify strong reflectiveness, allowing for potential insights into gender-related differences.

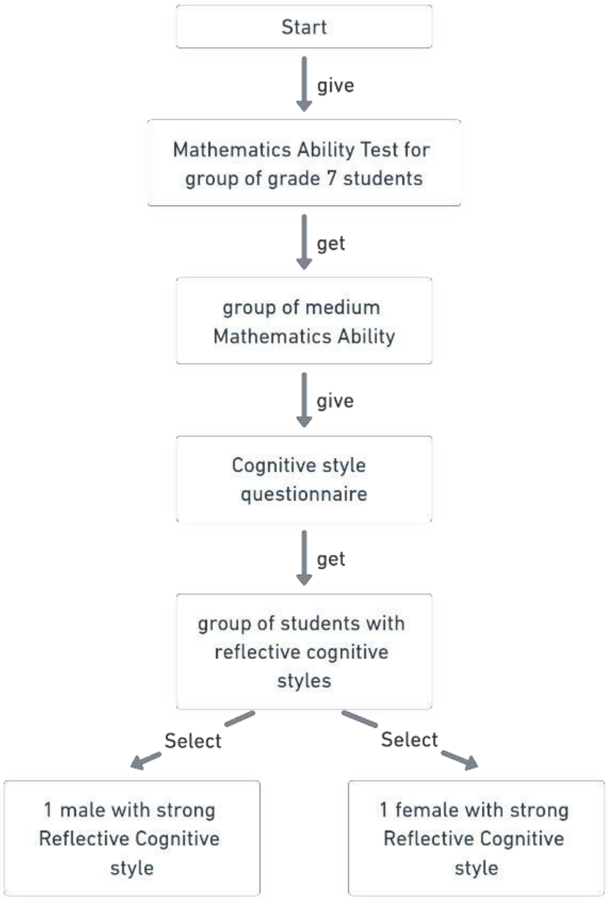

Figure 1. Participant Selection Process

Data Analysis Framework

In the Structuring stage (S), students organized the key elements of the source problem (S11) to understand its fundamental structure. They also organized and identified the main elements of the target problem (S12) to approach it with strategies similar to the source problem. This initial organization was crucial for setting a foundation upon which analogical reasoning could effectively occur. The Mapping stage (M) involved establishing correspondences between the source and target problems by finding similarities or relationships between their elements. As depicted in Figure 1, this process is visualized through arrows connecting elements like S11 to S12, illustrating direct relationships. Mapping enabled students to transfer knowledge from the familiar source problem to the less familiar target problem by highlighting analogous components. During the Applying stage (A), students formulated and tested strategies or solutions within the source problem (A11). They then adapted and implemented these effective strategies to the target problem (A12), making relevant adjustments as necessary. This stage reflected the practical application of the analogical connections established during mapping, allowing students to solve the target problem using insights gained from the source problem. In the Verifying stage (V), students checked the accuracy and completeness of the solution to the source problem (V11) and ensured that the solution to the target problem (V12) was correct and met the problem's objectives. This involved reviewing the entire problem-solving process to identify and correct any errors or omissions. Verification reinforced the reliability of their solutions and solidified their understanding of the problems. By employing this four-stage framework, we systematically examined how reflective students engage in analogical reasoning during mathematical problem-solving. This approach provided valuable insights into their cognitive strategies and highlighted potential areas for instructional support.

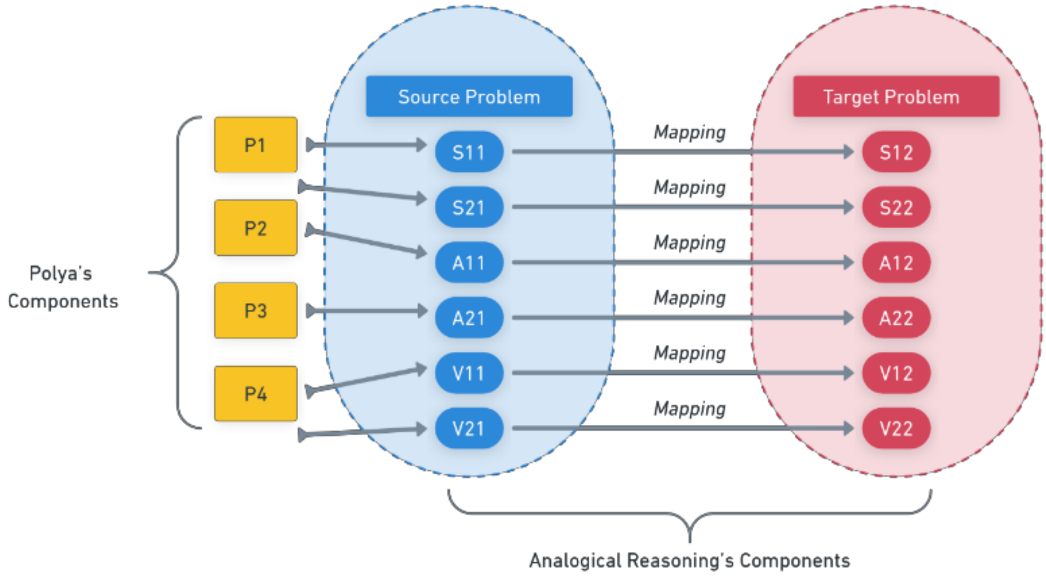

Figure 2. Diagram illustrating the stages of analogical reasoning between source and target problems

Ethical Considerations

Ethical approval for the study was obtained from Surabaya State University (Approval No. B/90483/UN38.3/LT.02.02/2024). The research adhered to ethical standards involving human participants. Participants and their parents or guardians were informed about the study's purpose, procedures, potential risks, and their rights, and written consent was obtained from all parties involved. Participants' identities were protected using pseudonyms (e.g., "Alex" and "Taylor"), and data were stored securely on password-protected devices accessible only to the researcher. They were also informed of their right to withdraw from the study at any time without any negative consequences.

RESULTS

This section presents the findings of the study, structured according to the stages of Structuring (S), Applying (A), and Verifying (V), which are mapped through the Mapping (M) process from the source problem to the target problem, as outlined in the Data Analysis Framework. The results focus on the two main participants, Participant S0713 (female) and Participant S0709 (male), both of whom exhibited strong reflective cognitive styles and medium mathematical ability.

Participant Selection and Characteristics

As detailed in the Methodology, the initial mathematics ability test was administered to 15 Grade 7 students (8 males and 7 females) at Chung Chung Christian School. Scores ranged from 70.0% to 96.0%. Nine students were classified as having medium mathematical ability (scores between 61.0% and 79.9%), and six were categorized as high ability (scores of 80.0% or above). From the medium ability group, five students with the highest scores (75.0% to 78.9%) were selected for further analysis. The Matching Familiar Figures Test (MFFT) identified four students (two females and two males) with reflective cognitive styles. The highest-scoring male and female reflective students, Participant S0713 and Participant S0709, were selected as the main participants to explore potential gender perspectives in the mapping process.

Table 1. Summary of Selected Students' Mathematics Scores and Cognitive Styles

Participant ID	Gender	Mathematics Score (%)	Cognitive Style	Reflectiveness Score
S0713	Female	**78.9**	**Reflective**	**Highest**
S0701	Female	78.5	Reflective	High
S0705	Female	76.3	Reflective	Moderate
S0709	Male	**77.2**	**Reflective**	**Highest**
S0710	Male	75.6	Impulsive	Low

Research Question 1: How do reflective middle school students perform the mapping process between source and target problems in mathematical analogical reasoning?

Table 2. Analysis of Structuring, Applying, and Verifying on Source and Target Problems with Mapping Check for Participant S0713 (Female)

Stage	Analysis of Source Problem	Analysis of Target Problem	Mapping Check between Source and Target Problems
Structuring (S)	➢ **Understanding the Source Problem**: Calculated the area of a rectangle. ➢ **Key Elements Identified**: Length (5 cm), Width (8 cm), and Area Formula (A= Length x Width) ➢ **Students' Note**: Length = 5 cm, Width = 8 cm; Area formula = length × width.	➢ **Understanding the Target Problem**: Calculated the area of a parallelogram. ➢ **Key Elements Identified**: Base (5 cm), Height (8 cm), and Area Formula (Base x Height) ➢ **Students' Note**: Parallelogram base = 5 cm, height = 8 cm; similar to rectangle but with slanted sides.	➢ **Elements Mapped**: Length (Source) ↔ Base (Target) ➢ Width (Source) ↔ Height (Target) ➢ **Conclusion**: The identified elements in the source problem are accurately mapped to the corresponding elements in the target problem, facilitating the application of the same area formula.
Applying (A)	➢ **Applying to Source Problem**: Calculated the area of the rectangle using the formula:	➢ **Applying to Target Problem**: Applied the same area formula to the parallelogram:	➢ **Strategies Applied**: Utilized the same multiplication strategy from the source problem.

Stage	Analysis of Source Problem	Analysis of Target Problem	Mapping Check between Source and Target Problems
	➢ $5\ cm \times 8\ cm = 40\ cm^2$	➢ $5\ cm \times 8\ cm = 40\ cm^2$	➢ **Conclusion**: The consistent application of the area formula across both problems validates the effectiveness of the mapping, resulting in accurate and consistent area calculations.
Verifying (V)	➢ Verifying Source Solution: Reviewed the calculation steps for the rectangle's area to ensure accuracy. ➢ Reflection: "Calculations are correct; units are squared centimeters."	➢ Verifying Target Solution: Compared the area results of the rectangle and parallelogram to ensure consistency. ➢ Reflection: "Since both areas are 40 cm², the mapping is correct."	➢ Verification Process: Conducted parallel verification for both source and target solutions. Conclusion: The identical area results confirm that the mapping between the source and target problems was accurately executed, ensuring ➢ the reliability of the applied strategies.

Table 3. Analysis of Structuring, Applying, and Verifying on Source and Target Problems with Mapping Check for Participant S0709 (Male)

Stage	Analysis of Source Problem	Analysis of Target Problem	Mapping Check between Source and Target Problems
Structuring (S)	➢ **Understanding the Source Problem**: Calculated the area of a rectangle. ➢ **Key Elements Identified**: Length (6 cm), Width (9 cm), and Area Formula (A= Length x Width) ➢ **Students' Note**: Length = 5 cm, Width = 8 cm; Area formula = length × width.	➢ **Understanding the Target Problem**: Calculated the area of a parallelogram. ➢ **Key Elements Identified**: Base (6 cm), Height (9 cm), and Area Formula (Base x Height) ➢ **Students' Note**: Parallelogram base = 5 cm, height = 8 cm; similar to rectangle but with slanted sides.	➢ **Elements Mapped**: Length (Source) ↔ Base (Target) ➢ Width (Source) ↔ Height (Target) ➢ **Conclusion**: Conclusion: The elements from the source problem are appropriately apped to the target problem, enabling the adaptation of the area formula to suit the different shapes.
Applying (A)	➢ **Applying to Source Problem**: Calculated the area of the rectangle using the formula: ➢ $6\ cm \times 9\ cm = 54\ cm^2$	➢ **Applying to Target Problem**: Applied the same area formula to the parallelogram: ➢ $6\ cm \times 9\ cm = 54\ cm^2$	➢ **Strategies Applied**: Utilized the same multiplication strategy from the source problem. ➢ **Conclusion**: The consistent application of the area formula across both problems validates the effectiveness of the mapping, resulting in accurate and consistent area calculations.
Verifying (V)	➢ Verifying Source Solution: Doublechecked Computations using a	➢ Verifying Target Solution: Compared the area results of the rectangle and	➢ Verification Process: Conducted parallel verification

Stage	Analysis of Source Problem	Analysis of Target Problem	Mapping Check between Source and Target Problems
	calculator to confirm the rectangle's area. ➢ Reflection: 54 cm² is accurate for the rectangle	parallelogram to ensure consistency. ➢ Reflection: "Since both areas are 54 cm², the mapping is correct."	for both source and target solutions. ➢ Conclusion: The identical area results confirm that the mapping between the source and target problems was accurately executed, ensuring the reliability of the applied strategies.

Research Question 2: What patterns emerge during this mapping process?

Table 4. Patterns Emerging During the Mapping Process

No	Pattern	Description
1	Integration of Mapping Throughout Problem-Solving Stages	Both participants consistently integrated the mapping process at each stage: • Structuring: Identified correspondences between elements of source and target problems. • Applying: Utilized mappings to apply strategies effectively. • Verifying: Confirmed the validity of their solutions through the lens of the mapping.
2	Depth of Structural Understanding	Demonstrated a strong grasp of underlying mathematical principles, enabling effective mapping and transfer of strategies. • Focused on structural similarities rather than superficial features, aligning with theories of analogical reasoning that emphasize relational correspondences (Gentner, 1983).
3	Reflective Cognitive Engagement	Exhibited high levels of metacognition, frequently self-monitoring and reflecting on their problem-solving processes. • Took deliberate steps to ensure comprehension before proceeding, indicative of reflective thinking.
4	Modality Preferences and Mapping	**Participant S0713:** Preferred verbal reasoning and written explanations, enhancing understanding through detailed notes and annotations. **Participant S0709:** Favored visual-spatial reasoning, using drawings and diagrams extensively to facilitate mapping.
5	Adaptability and Strategy Transfer	Successfully adapted strategies from the source problem to the target problem, making necessary adjustments for different shapes. • Their reflective nature facilitated flexible thinking and problem-solving adaptability.
6	Importance of Verification	Emphasized the verification stage as crucial for confirming accuracy. • Both participants revisited previous steps if inconsistencies were found, showcasing thoroughness and attention to detail.

CONCLUSION

This study provides a nuanced examination of the mapping processes in analogical reasoning among reflective middle school mathematics students. By integrating the mapping process throughout the

Structuring, Applying, and Verifying stages, the participants demonstrated effective transfer and adaptation of mathematical strategies across different problem contexts. The identified patterns highlight the critical role of structural understanding, reflective cognitive engagement, modality preferences, adaptability, and thorough verification in enhancing mathematical problem-solving skills. These insights underscore the potential of analogical reasoning as a powerful tool in mathematics education and advocate for instructional strategies that foster deep structural comprehension and accommodate diverse cognitive styles. Despite its limitations, this study contributes to the growing body of literature on cognitive strategies in mathematical learning and lays the groundwork for future research aimed at optimizing educational practices to support diverse learner needs.

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